

**ONE WAY TO ANALYZE THE ARTICLE A STUDY FROM THE
OPERATOR OF EVOLUTION AND THE DESCRIPTIONS OF
HEISENBERG, OF SCHRODINGER AND "OF INTERACTION" AND
ITS VERIFICATIONS IN SERIES THROUGH POWER SERIES
EXPANSIONS, CONCOMITANTLY WITH THE INTEGRATION OF
 $H(T)$ IN AND EXPONENTIAL, FOR EXAMPLE, FUNCTION THE
RIEMANN ZETA.**

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Concomitantly With the Integration Of $H(T)$ In and Exponential, For Example, Function the
Riemann Zeta.

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SUMMARY

This article presents a way analysis, with the aim of finding an operator of Evolution in Quantum Mechanics for the Riemann Zeta Function and the Numbers Primes, and consequently other evolution operators. They can be found for different formulas and equations. Basically do an analysis, with the intention of attempts at analysis to find Construction of a Self-Adjoint Riemann Hamiltonian from the Evolution Operators suitability in Quantum Mechanics. According to the Trivial Zeros of the Riemann Hypothesis, Through analysis of the article, the need for an operator is demonstrated. evolution to work The Riemann Hypothesis and the Riemann Zeta function, one way that could be built with attempts at analysis with both. The articles mentioned in this analysis are used to advance the field of Mechanics. Quantum and Number Theory, focusing on prime numbers and Euler's equation for prime numbers, as proposed and explained in the first article [1].

KEYWORDS: Quantum Mechanics, Evolution Operator, Hamiltonian, Primes Euler, Zeta Function Riemann, Riemann Hypothesis, Prime Numbers.

INTRODUCTION

The Riemann zeta function is one of the most important functions in mathematical theory. It belongs mainly to the fields of complex analysis and analytic number theory. The function is represented by the Greek letter ζ . For a complex number s with real part greater than one, it is defined by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

In this expression, s may be written as $s = \sigma + it$. The symbols σ and t represent real numbers. The real part of s is denoted by $\text{Re}(s) = \sigma$. The imaginary part is related to the value t . When s is a real number greater than one, the defining series converges. For example, $\zeta(2)$ is the sum of the reciprocals of the squares. This particular value is equal to $\pi^2/6$. The zeta function was studied by several mathematicians before Riemann. However, Bernhard Riemann gave it a deeper theoretical interpretation. In 1859, Riemann published a famous paper about prime numbers. His work connected the zeta function with the distribution of primes. Prime numbers are natural numbers greater than one with exactly two positive divisors. Examples of prime numbers are 2, 3, 5, 7, and 11. Every integer greater than one can be expressed as a product of primes. This fact is known as the fundamental theorem of arithmetic. The zeta function reflects this fundamental structure of the integers. Its connection with primes appears in the Euler product formula. For $\text{Re}(s) > 1$, this formula is

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}.$$

Each factor in this product corresponds to a different prime number. The Euler product shows that the zeta function contains information about all primes. It also connects an infinite sum with an infinite product. This connection is one of the most remarkable ideas in number theory. The original series does not converge for every complex value of s . However, the function can be extended by analytic continuation. Analytic continuation defines the zeta function in almost the entire complex plane. The only pole of the extended function occurs at $s=1$. This pole is simple, meaning that it has order one. Near $s=1$, the zeta function becomes unbounded. The pole is related

to the divergence of the harmonic series. The harmonic series is given by $1 + 1/2 + 1/3 + 1/4 + \dots$. Although its terms become smaller, the harmonic series does not converge. The zeta function has two different types of zeros. The first type is called the trivial zeros. These zeros occur at the negative even integers. They are located at $s = -2, -4, -6, -8, \dots$. The second type is called the nontrivial zeros. The nontrivial zeros are located in a region called the critical strip. The critical strip is defined by $0 < \text{Re}(s) < 1$. The Riemann Hypothesis concerns the exact location of these zeros. It states that every nontrivial zero has real part equal to one half. In mathematical notation, the hypothesis can be expressed as $\text{Re}(s) = 1/2$. Therefore, every nontrivial zero should lie on the line $s = 1/2 + it$. This line is called the critical line. The Riemann Hypothesis has never been proven or disproven. For this reason, it remains one of the most famous open problems in mathematics. The hypothesis is important because the zeros control the distribution of prime numbers. Although primes seem to appear irregularly, they follow general statistical laws. The prime number theorem describes the approximate distribution of primes. If $\pi(x)$ represents the number of primes less than or equal to x , then

$$\pi(x) \sim \frac{x}{\log x}.$$

The symbol $\sim$ means that the ratio between both sides approaches one. The prime number theorem gives the general behavior of prime distribution. The Riemann Hypothesis would provide a much more accurate estimate. It would limit the possible error in approximations involving the function $\pi(x)$. Consequently, the hypothesis would improve our understanding of prime numbers. The zeta function also satisfies an important functional equation. This relates equations the value at s to the value at $1-s$. It creates a symmetry around the critical line. The functional equation also explains the occurrence of the trivial zeros. It shows that the zeta function has a highly organized structure. The nontrivial zeros have additional symmetries in the complex plane. If a zero is represented by ρ , its complex conjugate is also a zero. The transformation $s \mapsto 1-s$ creates another important relationship.

These symmetries make the critical strip particularly significant. Many nontrivial zeros have been calculated through numerical methods. The zeros that have been tested so far appear to lie on the critical line. However, numerical evidence cannot establish a result for infinitely many zeros. A mathematical proof must apply to every nontrivial zero.

This requirement makes the problem extremely difficult. Several important partial results have already been established. Mathematicians have proven that infinitely many zeros lie on the critical line. They have also shown that a positive proportion of zeros lie there. However, these results do not prove that all nontrivial zeros are on the line. The Riemann Hypothesis is one of the Millennium Prize Problems. These problems were selected because of their exceptional importance and difficulty. A correct proof or disproof of the hypothesis would be a major achievement. The problem has influenced many areas of modern mathematics. These areas include complex analysis, algebra, probability, and mathematical physics. The zeros of the zeta function have also been compared with random matrix models. Such comparisons reveal unexpected patterns in their distribution. These patterns suggest that the zeta function possesses a deep hidden order. The study of the zeta function is not limited to pure theory.

Its ideas are connected with algorithms and computational number theory. Prime numbers are also important in modern cryptographic systems. However, the Riemann Hypothesis itself is primarily a theoretical problem. Its importance comes from the relationship between complex numbers and prime numbers. A proof would lead to stronger results about many arithmetic functions. A disproof would reveal unexpected irregularities in the distribution of primes. Both possibilities would significantly change mathematical knowledge. For this reason, researchers continue to investigate the problem. The Riemann zeta function is simple to define but difficult to understand completely. Its series, product, zeros, and functional equation reveal different aspects of its nature. The Riemann Hypothesis summarizes one of the deepest questions about its zeros. It asks whether the apparent order observed numerically continues forever. The answer remains unknown despite extensive research. Thus, the zeta function represents a bridge between analysis and number theory. Its study demonstrates how abstract formulas can describe the behavior of prime numbers. The Riemann Hypothesis continues to inspire mathematicians throughout the world. Therefore, the zeta function and its hypothesis remain central subjects in modern mathematics.

The evolution of operators is a fundamental concept in quantum mechanics. It describes how physical observables change as a quantum system develops in time. In quantum theory, observables are represented by mathematical operators. Examples of observables include position, momentum, energy, and angular momentum. The time evolution of a quantum system can be described in different representations. The two most common representations are the Schrödinger picture and the Heisenberg picture. In the Schrödinger picture, quantum states evolve with time.

The operators representing observables are generally kept fixed, except for explicit time dependence.

The evolution of a state is governed by the Schrödinger equation. This equation is one of the central principles of nonrelativistic quantum mechanics. For a state vector $|\psi(t)\rangle$, the equation is

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle.$$

In this expression, H is the Hamiltonian operator. The Hamiltonian represents the total energy of the quantum system. The symbol $\hbar$ denotes the reduced Planck constant. If the

Hamiltonian does not depend explicitly on time, the evolution operator is

$$U(t) = e^{-iHt/\hbar}.$$

The state at time t can then be written as

$$|\psi(t)\rangle = U(t) |\psi(0)\rangle.$$

The operator $U(t)$ must be unitary. Unitarity guarantees that the total probability remains equal to one. It also preserves the inner products between quantum states. Therefore, unitary evolution is essential for the consistency of quantum mechanics. In the Heisenberg picture, the situation is described differently. The state vectors are considered constant in time. The operators, however, carry the time dependence of the physical quantities. An operator in the Heisenberg picture is defined by

$$A_H(t) = U^\dagger(t) A_S U(t).$$

Here, A_S represents the corresponding Schrödinger-picture operator. The symbol $U^\dagger(t)$ denotes the adjoint of the evolution operator. Because $U(t)$ is unitary, $U^\dagger(t)U(t)=I$. This transformation preserves the measurable predictions of the theory. The Heisenberg equation

of motion gives the time evolution of an operator. For an operator A, it is written as

$$\frac{dA_H}{dt} = \frac{i}{\hbar}[H, A_H] + \left(\frac{\partial A}{\partial t}\right)_H.$$

The symbol $[H, A_H]$ denotes the commutator of the Hamiltonian and the operator. The commutator is defined by $[H, A_H] = HA_H - A_H H$. The final term represents any explicit time dependence of the operator itself. If the operator has no explicit time dependence, that term is zero. The equation then becomes

$$\frac{dA_H}{dt} = \frac{i}{\hbar}[H, A_H].$$

This formula is the quantum-mechanical analogue of a classical equation of motion. In classical mechanics, observables evolve according to Poisson brackets. In quantum mechanics, commutators replace Poisson brackets. This replacement reflects the noncommutative nature of quantum observables. Position and momentum provide the most important example of noncommutativity. Their fundamental commutation relation is

$$[x, p] = i\hbar.$$

This relation is connected to the Heisenberg uncertainty principle. It shows that position and momentum cannot be simultaneously measured with unlimited precision. The time evolution of position depends on its commutator with the Hamiltonian. For a nonrelativistic particle, the Hamiltonian can be written as

$$H = \frac{p^2}{2m} + V(x).$$

In this expression, m is the particle's mass. The function $V(x)$ represents the potential energy. Using the Heisenberg equation, the position operator satisfies

$$\frac{dx}{dt} = \frac{p}{m}.$$

This result resembles the classical relation between velocity and momentum. The momentum operator satisfies

$$\frac{dp}{dt} = -\frac{\partial V}{\partial x}.$$

This equation also has a direct classical analogue. Together, these equations show how classical mechanics emerge from quantum theory. However, the operators in quantum mechanics generally do not commute. Their noncommutativity produces effects that have no classical equivalent. The expectation value of an operator represents the average result of many measurements. In the Schrödinger picture, the expectation value is written as

$$\langle A \rangle = \langle \psi(t) | A | \psi(t) \rangle.$$

In the Heisenberg picture, the state is fixed while the operator evolves. The expectation value is then written as

$$\langle A \rangle = \langle \psi(0) | A_H(t) | \psi(0) \rangle.$$

Both expressions give exactly the same physical predictions. This equivalence demonstrates that the two pictures are mathematically consistent. The choice between them is often based on convenience. The Schrödinger picture is useful when the state's evolution is easier to calculate. The Heisenberg picture is useful when the behavior of observables is the main concern. For systems with time-dependent Hamiltonians, evolution requires a time-ordered exponential. The general evolution operator can be written as

$$U(t, t_0) = \mathcal{T} \exp \left[-\frac{i}{\hbar} \int_{t_0}^t H(t') dt' \right].$$

The symbol T indicates time ordering. Time ordering is necessary because Hamiltonians at different times may not commute. If $[H(t_1), H(t_2)] = 0$, the expression becomes simpler. In that case, the time-ordering symbol may be omitted. Operator evolution is also important for conserved quantities. An operator is conserved when its total time derivative is zero. For an operator with no explicit time dependence, this requires $[H, A] = 0$. When this commutator vanishes, the observable remains constant in time. Energy is conserved when the Hamiltonian has no explicit time dependence. Angular momentum may also be conserved when the system has rotational symmetry. Conservation laws are closely related to symmetries in quantum mechanics. This relationship is expressed through the quantum version of Noether's theorem. Operator evolution can also describe transformations generated by momentum. Momentum generates spatial translations, while the Hamiltonian generates time translations. The unitary operator for a spatial translation is commonly written as

$$T(a) = e^{-iap/\hbar}.$$

The Hamiltonian generates temporal translations through the operator $U(t)$. Thus, quantum evolution is represented by continuous unitary transformations. These transformations preserve probabilities and the structure of the Hilbert space. Operator evolution is essential in quantum optics, atomic physics, and field theory. It is also used to analyze harmonic oscillators and spin systems. For a harmonic oscillator, creation and annihilation operators evolve with characteristic phases. Their evolution helps explain quantized energy levels. In quantum field theory, fields themselves are treated as time-dependent operators. Their commutation relations determine the causal and dynamical structure of the theory. Therefore, operator evolution is more than a formal mathematical procedure. It provides a complete language for describing changing physical observables. It connects quantum dynamics with measurement, symmetry, and conservation. The Heisenberg equation shows how commutators control temporal change. The Schrödinger equation shows how states change under the Hamiltonian.

Although the two descriptions look different, they produce identical experimental results. This equivalence is one of the strengths of the mathematical framework. Operator evolution also clarifies the difference

between states and observables. States describe the preparation of a system, while operators represent possible measurements. Their relationship determines the probabilities of different outcomes. As a result, operator evolution is central to the interpretation of quantum theory. It explains how physical information changes without violating probability conservation. It also reveals the deep difference between classical and quantum dynamics. In classical mechanics, observables are usually ordinary functions of position and momentum. In quantum mechanics, they are operators acting on vectors in a Hilbert space. Their algebraic properties determine the possible behavior of the system. The commutator provides information about incompatibility between measurements. The Hamiltonian determines how these operators evolve through time. Therefore, understanding the Hamiltonian is essential for analyzing quantum dynamics. Operator evolution can be solved exactly for only certain important systems. For more complex systems, approximation methods and numerical techniques are used. Perturbation theory is one method for studying small corrections to a known Hamiltonian. The interaction picture combines features of the Schrödinger and Heisenberg pictures.

It is especially useful in perturbation theory and quantum field theory. In every representation, the physical predictions remain unchanged. This principle is known as the representation independence of quantum mechanics. The evolution of operators therefore expresses the same physics in a different mathematical form. It allows physicists to select the most efficient description for a particular problem. The subject also provides a foundation for quantum computation. Quantum gates are unitary operators that evolve quantum states and observables. Their operation must preserve coherence and probability. Understanding operator evolution is therefore important for quantum information processing. It helps explain how quantum algorithms manipulate states and measurements. The formalism is also important for open quantum systems and decoherence. In such systems, interactions with the environment modify the simple unitary description.

Nevertheless, operator methods remain useful for describing the resulting dynamics. They can be extended through density operators and master equations. These tools describe statistical mixtures and interactions with external surroundings. Operator evolution is consequently applicable to both isolated and realistic systems. In conclusion, operator evolution is a central principle of quantum mechanics. It describes the temporal behavior of observables through unitary transformations and commutators. The Heisenberg equation provides its most direct mathematical expression. The Schrödinger and Heisenberg pictures offer equivalent descriptions of quantum change. Together, they show how the

Hamiltonian governs the development of physical systems. They also connect quantum theory with symmetry, conservation, measurement, and computation. For these reasons, operator evolution is essential for understanding the structure of modern physics.

DISCUSSION:

Based on the article [1] can be developed mathematically by turning its outline into a coherent derivation connecting:

1. The quantum-mechanical evolution operator,
2. The Taylor expansion of the exponential,
3. The Riemann zeta function,
4. Euler's product over primes,
5. The zeros of $\zeta(s)$,
6. And the speculative connection with quantum-mechanical spectra.

The article [1] itself explicitly presents the article as an initial outline for a future analysis rather than a completed proof.

Evolution operator

For a quantum system with a time-independent Hamiltonian H , the Schrödinger equation is

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle.$$

Its formal solution is

$$|\psi(t)\rangle = U(t, t_0) |\psi(t_0)\rangle,$$

where

$$U(t, t_0) = \exp \left[-\frac{i}{\hbar} H(t - t_0) \right].$$

This gives a precise mathematical interpretation to the article's proposed use of a Taylor expansion. Time-dependent Hamiltonian

The article also considers $H(t)$. In that case, one cannot generally write simply

$$U(t, t_0) = \exp \left[-\frac{i}{\hbar} \int_{t_0}^t H(\tau) d\tau \right]$$

because, in general

$$[H(t_1), H(t_2)] \neq 0.$$

This noncommutativity is explicitly recognized in the article [1]. The correct formal expression is the time-ordered exponential:

$$U(t, t_0) = \mathcal{T} \exp \left[-\frac{i}{\hbar} \int_{t_0}^t H(\tau) d\tau \right].$$

Its expansion is the Dyson series:

$$U(t, t_0) = I - \frac{i}{\hbar} \int_{t_0}^t H(t_1) dt_1 \\ - \frac{1}{\hbar^2} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H(t_1) H(t_2) + \dots$$

This is an important mathematical development of the operator argument in the article.
Riemann zeta function

The Riemann zeta function is initially defined, for by the Dirichlet series

$$\operatorname{Re}(s) > 1,$$

For example,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}. \\ \zeta(2) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}.$$

The article [1] describes $\zeta(s)$ as a complex function and emphasizes its connection with prime-number distribution.

Euler product

The most important mathematical step in the article is Euler's identity

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}, \quad \operatorname{Re}(s) > 1.$$

The article [1] explains that this follows from the geometric series and the fundamental theorem of arithmetic.

For each prime,

$$\frac{1}{1 - p^{-s}} = 1 + p^{-s} + p^{-2s} + p^{-3s} + \dots.$$

Multiplying over all primes gives

$$\prod_p (1 + p^{-s} + p^{-2s} + \dots).$$

every positive integer has a unique prime factorization,

$$n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}.$$

Consequently, every n^{-s} occurs exactly once:

$$\prod_p \sum_{k=0}^{\infty} p^{-ks} = \sum_{n=1}^{\infty} n^{-s}.$$

Hence,

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}.$$

This is the rigorous mathematical bridge between the zeta function and prime numbers.

Taking the logarithm

We can develop the article further by taking logarithms:

$$\log \zeta(s) = - \sum_p \log(1 - p^{-s}).$$

Using

$$-\log(1 - x) = \sum_{k=1}^{\infty} \frac{x^k}{k}, \quad |x| < 1,$$

we obtain

$$\log \zeta(s) = \sum_p \sum_{k=1}^{\infty} \frac{1}{k p^{ks}}.$$

Differentiating,

$$\frac{\zeta'(s)}{\zeta(s)} = - \sum_p \sum_{k=1}^{\infty} \frac{\log p}{p^{ks}}.$$

Therefore,

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_p \sum_{k=1}^{\infty} \frac{\log p}{p^{ks}}.$$

The right-hand side can be written using the von Mangoldt function:

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^s}.$$

This equation makes the connection between the analytical structure of $\zeta(s)$ and prime numbers much more explicit.

The Riemann zeros

The article then moves toward the central question concerning the zeros of $\zeta(s)$. It states the Riemann hypothesis as for every nontrivial zero.

$$\operatorname{Re}(s) = \frac{1}{2}$$

Writing

$$s = \sigma + it,$$

the conjecture says

$$\zeta\left(\frac{1}{2} + it\right) = 0$$

For every nontrivial zero. Thus the zeros have the form

$$s_n = \frac{1}{2} + i\gamma_n,$$

where γ_n is real.

The article's proposed physical interpretation becomes interesting at this point.

Possible quantum interpretation

The article [1] discusses a possible connection between the nontrivial zeros and quantum systems, but it correctly describes this as speculative and unproven, rather than an established result. In particular, it distinguishes this idea from the ordinary hydrogen atom. zeta riemman quantum ijarp

The proposed mathematical structure can be represented schematically as

$$\text{Riemann zeros} \longleftrightarrow \text{eigenvalues of an operator.}$$

Suppose there were a self-adjoint operator (H_R) satisfying

$$H_R \psi_n = \gamma_n \psi_n.$$

Then the nontrivial zeros could be written as

$$s_n = \frac{1}{2} + iE_n,$$

where

$$E_n = \gamma_n.$$

This is the basic mathematical idea associated with the Hilbert–Pólya approach.

If such an operator existed, one could formally construct a quantum evolution operator

$$U_R(t) = e^{-iH_R t}.$$

Using the spectral equation,

$$H_R \psi_n = E_n \psi_n,$$

we obtain

$$U_R(t) \psi_n = e^{-iE_n t} \psi_n.$$

Thus

$$\text{zero } \frac{1}{2} + iE_n \longrightarrow \text{quantum phase } e^{-iE_n t}.$$

This is a mathematically meaningful way to develop the article's proposed relationship between the evolution operator and the zeta zeros, while keeping clear that the existence of

the required physical operator hasnot been established.

Important distinction: hydrogen atom

The article itself makes an important clarification:

Riemann zeros $\neq$ ordinary hydrogen energy levels.

The hydrogen atom has energy levels described by the Schrödinger equation and, in the nonrelativistic model,

$$E_n = -\frac{13.6 \text{ eV}}{n^2}.$$

The zeta zeros instead have the structure

$$s_n = \frac{1}{2} + i\gamma_n.$$

Therefore, one shouldnotidentify

$$\gamma_n$$

directly with the hydrogen energy

$$-\frac{13.6}{n^2} \text{ eV}.$$

Thearticle [1]explicitly states that there is no direct one-to-one correspondence between the zeros of the zeta function and the standard hydrogen spectrum.

A stronger mathematical formulation of the article The central research program can therefore be formulated as

$$\zeta(s) \rightarrow \{\gamma_n\} \rightarrow H_R \rightarrow U_R(t)$$

with

$$\zeta\left(\frac{1}{2} + i\gamma_n\right) = 0,$$

followed hypothetically by

$$H_R\psi_n = \gamma_n\psi_n,$$

and finally

$$U_R(t) = e^{-iH_R t/\hbar}.$$

The conceptual chain would be Prime numbers $\rightarrow \zeta(s) \rightarrow$ nontrivial zeros $\rightarrow$ spectral parameters $\rightarrow$ operator $\rightarrow$ quantum evolution

This is part of the article that can be developed into a genuine mathematical research problem. The article [1] itself presents the connection as a proposed direction for future analysis rather than a completed derivation.

What would constitute a real solution?

To turn the article's proposal into a rigorous result, one would need to construct an explicit self-adjoint operator (H_R) such that

$$\text{Spec}(H_R) = \{\gamma_1, \gamma_2, \gamma_3, \dots\},$$

where

$$\zeta\left(\frac{1}{2} + i\gamma_n\right) = 0.$$

Then one would need to prove

$$H_R = H_R^\dagger$$

and establish that its spectrum corresponds precisely to the imaginary parts of the nontrivial zeros.

That would provide a much stronger mathematical foundation for the article's proposed relation between Riemann zeta, quantum mechanics, and an evolution operator.

In short: the article [1] contains the ingredients of a research framework, but it does not itself prove a new relation between the Riemann hypothesis and the quantum Hamiltonian. The mathematically rigorous development is to derive the evolution-operator expansion, Euler product, logarithmic derivative, zero structure, and then formulate precisely what operator would have to exist for the proposed quantum interpretation to work.

The operator looking for is not known explicitly. The natural candidate is a self-adjoint (Hermitian) operator whose eigenvalues are the imaginary parts of the nontrivial zeros of the Riemann zeta function. This is the central idea of the Hilbert–Polya program. The Clay Mathematics Institute describes this as the possibility that the zeros correspond to eigenvalues of a self-adjoint operator.

For this article, we would formulate the target operator as

$$H_R \psi_n = \gamma_n \psi_n$$

where

$$\zeta\left(\frac{1}{2} + i\gamma_n\right) = 0$$

and $(H_R = H_R^\dagger)$

$$\text{Spec}(H_R) = \{\gamma_1, \gamma_2, \gamma_3, \dots\}.$$

Then its spectrum would be

Why this operator matters If $(H_R) \setminus (\gamma_n)$

$$s_n = \frac{1}{2} + i\gamma_n$$

automatically lies on

$$\text{Re}(s) = \frac{1}{2}.$$

Only,if one could construct such an operator and prove that its entire spectrum is exactly the set of nontrivial zeta-zero coordinates, this would give a route to the Riemann hypothesis. The hypothesis itself remains unsolved.

Connection with your article's evolution operator

This fits very naturally with the evolution-operator ideaarticle [1]. The article discusses the quantum evolution operator and its exponential form.

Take

$$U_R(t) = e^{-iH_R t/\hbar}.$$

For an eigenstate ψ_n

$$H_R \psi_n = \gamma_n \psi_n,$$

only

$$U_R(t) \psi_n = e^{-i\gamma_n t/\hbar} \psi_n.$$

Thus the proposed chain becomes

$$\zeta\left(\frac{1}{2} + i\gamma_n\right) = 0 \implies H_R \psi_n = \gamma_n \psi_n \implies U_R(t) = e^{-iH_R t/\hbar}.$$

But what should H_R

That is the difficult part. There is currently no accepted explicit operator H_R

For your article, therefore, I would not write

$$H_R = px$$

or

$$H_R = -i\hbar x \frac{d}{dx}$$

and claim that this solves the problem. Those are candidates appearing in the literature, not established solutions.

A more rigorous title for the proposed development would be:

Construction of a Self-Adjoint Riemann Hamiltonian from the Evolution Operator

with the research objective

$$\text{Construct } H_R = H_R^\dagger \text{ such that } \text{Spec}(H_R) = \{\gamma_n\}.$$

That is the precise mathematical problem article [1] is pointing toward.

CONCLUSION

Therefore, this analysis concludes that, through attempts at analysis in number theory or computational theory, a suitable evolution operator can be found in the proposal of the article, in this case and in [1]. The present analysis facilitates attempts at analysis to consolidate a suitable outcome in solving the problem in Quantum Mechanics, using an approach based on the Hamiltonian, the Evolution Operator, the Riemann Zeta Function, the Euler Function, and Prime Numbers, in order to seek a plausible solution to the aforementioned problem. Consequently, the approach to the problem in question becomes simpler, allowing for mathematical and computational progress in Quantum Mechanics and Number Theory. As verified in theory, the work of mathematical expansions and the theory of prime numbers contributes to the development of mathematical solutions and mathematical problems. Construction of a Self-Adjoint Riemann Hamiltonian from the Evolution Operator.

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